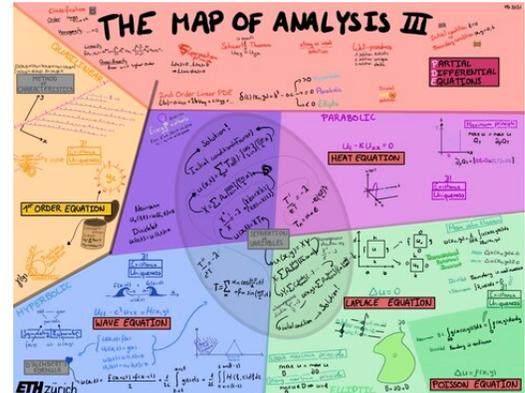


PVK2020_Ana3

1. Did you already have a look at the map of Analysis 3?

0 POINTS

- True
- False



2. A solution must be at least C^2 in order to be well-posed.

1 POINT

- True
- False

3. State the order and the linearity/nonlinearity of the following PDE:

$$u_{xy} \cos(u_{yy}) + u_{xyx}u_{xxy} + u_x u_{xxy} = e^\pi u^{1.5}$$

1 POINT

- A 3rd Order nonlinear
- B 2nd Order quasilinear
- C 3rd Order quasilinear
- D 2nd Order nonlinear
- E 2nd Order linear
- F 3rd Order linear

4.

$$x_t(t, s) = y, \quad y_t(t, s) = -x, \quad u_t(t, s) = 0, \\ x(0, s) = s, \quad y(0, s) = 0, \quad u(0, s) = g(s^2), \quad \text{where } s > 0.$$

You're given the following system of ODEs. How could you solve it?

1 POINT

- (A) Go to the next question
- (B) Differentiate the equation for x w.r.t. s
- (C) Differentiate the equation for y w.r.t. t
- (D) Guess

5. You're given the initial condition $u(1, y) = f(y)$ for $y > 0$. How do you represent this as an initial curve?

1 POINT

- (A) $(s, 1, f(s)), s > 0$
- (B) $(s, s, f(s)), s > 0$
- (C) $(1, s, f(s)), s > 0$
- (D) All of the above

6. How can you parametrise the unit circle?

1 POINT

- (A) $(\cos(s), \sin(s)), 0 < s \leq \pi$
- (B) $(\cos(s), \sin(s)), 0 < s \leq 2\pi$
- (C) (s, s^2)
- (D) $(\cos(s), \sin(s)), -\pi < s \leq \pi$

7. Is

$$x_0 = s^2, y_0 = 3, \bar{u}(0, s) = 1 + s^2$$

a valid parametrization of the initial condition

$$u(x, 3) = 1 + x ?$$

1 POINT

- (T) True
- (F) False

8. Consider the conservation law

$$\begin{cases} u_y + (u^3 + 1)u_x = 0, & x \in \mathbb{R}, y > 0, \\ u(x, 0) = h(x), & x \in \mathbb{R}, \end{cases}$$

where $h(x) = 1$ if $x \leq 0$, $h(x) = (1 - x)^{\frac{1}{3}}$ if $0 < x < 1$ and $h(x) = 0$ if $x \geq 1$.

Compute the critical time y_c

1 POINT

- (A) 2
 (B) 0
 (C) -1
 (D) 1

9.
 • *hyperbolic* if $\delta(L)(x_0, y_0) > 0$
 • *parabolic* if $\delta(L)(x_0, y_0) = 0$
 • *elliptic* if $\delta(L)(x_0, y_0) < 0$

Consider the PDE $x^2 u_{xx} + xy u_{xy} - y u_{yy} + \sin(x^2) u_x + 1 = 0$. Which of the following statements are correct:

1 POINT

- (A) Parabolic in $\Omega = \{y = 4, x \in \mathbb{R}\}$
 (B) Parabolic for $\Omega = \{x = 0, y \in \mathbb{R}\}$
 (C) Elliptic for $\Omega = \{x < 0, -3 < y < 0\}$
 (D) Hyperbolic for $\Omega = \{y > 0, x \in \mathbb{R}\}$

10. (f) Let $u : D \rightarrow \mathbb{R}$ be the solution to the Laplace equation with Neumann boundary conditions

$$\begin{cases} \Delta u = 0 & \text{in } D, \\ \partial_\nu u = \alpha x^4 + \beta y^3 & \text{on } \partial D, \end{cases}$$

where $D = \{x^2 + y^2 < 1\}$. For which values α and β is the previous problem solvable?

For which values of α, β is the problem solvable?

1 POINT

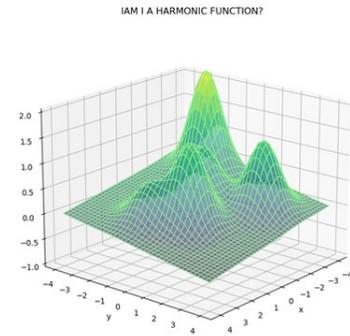
- (A) For all $\alpha, \beta \in \mathbb{R}$, s.t. $\alpha = 0$
 (B) For all $\alpha, \beta \in \mathbb{R}$, s.t. $\beta = 0$
 (C) For all $\alpha, \beta \in \mathbb{R}$, s.t. $4\alpha + 3\beta = 0$
 (D) For all $\alpha, \beta \in \mathbb{R}$, s.t. $3\alpha + 4\beta = 0$

11. This winter you're going skiing, on your way there in the train you decide to relax in the train by studying a little bit of analysis 3. As you see the mountains from a far, you wonder:

"Do mountains represent harmonic function?"

1 POINT

- True
 False

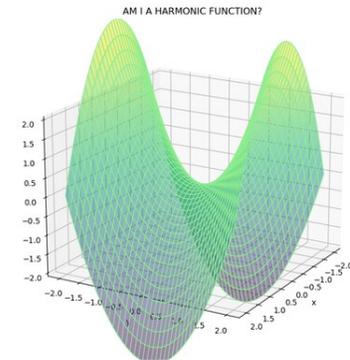


12. After some time you've made up your way to a pass, and wonder the same thing:

"Can the pass be parametrized by a harmonic function?"

1 POINT

- True
 False



13. (e) Consider the Dirichlet problem

$$\begin{cases} \Delta u = 0, & \text{in } D, \\ u = \frac{x}{x^2+y^2} & \text{on } \partial D, \end{cases}$$

where the domain D is the annulus defined by $D := \{(x, y) \in \mathbb{R}^2 : 1 < \sqrt{x^2 + y^2} < 2\}$.

What is the maximum of u ?

You've made your way up the highest mountain and in the far, you see to the Canton of St.Gallen! You notice that this Canton is basically an annulus around Appenzell and that you can crudely model it using the following equation.

You then ask yourself, what is the highest point in the Canton of St.Gallen? (in thousand meters)

1 POINT

- A 3/4
 B 1/4
 C 1
 D 3/2

SOLUTIONS

JEAN902

PVK2020_Ana3

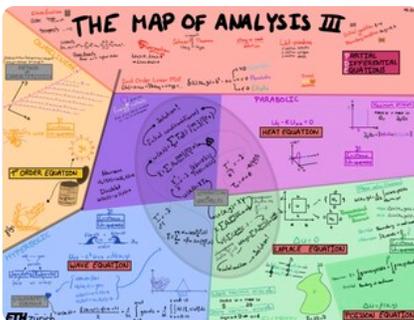


Save and Exit

Align Quiz to Standard

Share

1. Did you already have a look at the map of Analysis 3?



0 POINTS

True

- i** It's available here: https://n.ethz.ch/~megretj/download/Notizen/Map_of_Analysis_3.pdf

2. A solution must be at least C^2 in order to be well-posed.

1 POINT

False

- i** No. The conditions for **well-posedness** are existence, uniqueness and stability. However, being C^1 is sufficient. C^2 a.k.a. smoothness is necessary for a **strong** solution!



3. State the order and the linearity/nonlinearity of the following PDE:

$$u_{xxy} \cos(u_{yy}) + u_{xyx}u_{xxy} + u_x u_{xxy} = e^\pi u^{1.5}$$

1 POINT

- A 3rd Order nonlinear
- B 2nd Order quasilinear
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- E 2nd Order linear
- F 3rd Order linear
- i Clearly the equation is not linear w.r.t. u . The highest degree term is u_{xxy} (which is the same as u_{xyx}), it is of order 3. As this term is not linear (it's squared: $u_{xyx}u_{xxy}$) this equation is nonlinear.

4. You're given the following system of ODEs. How could you solve it?

$$\begin{aligned} x_t(t,s) &= y, & y_t(t,s) &= -x, & u_t(t,s) &= 0, \\ x(0,s) &= s, & y(0,s) &= 0, & u(0,s) &= g(s^2), \text{ where } s > 0. \end{aligned}$$

1 POINT

- A Go to the next question
- B Differentiate the equation for x w.r.t. s
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5. You're given the initial condition $u(1, y) = f(y)$ for $y > 0$. How do you represent this as an initial curve?

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a valid parametrization of the initial condition

$$u(x, 3) = 1 + x?$$

1 POINT

False

- i Be careful ! Now s only parametrizes $x > 0$ but we need to parametrize for all x ($x = s, y = 3, \bar{u}(0, s) = 1 + s$ would work).



8. Consider the conservation law

$$\begin{cases} u_y + (u^3 + 1)u_x = 0, & x \in \mathbb{R}, y > 0, \\ u(x, 0) = h(x), & x \in \mathbb{R}, \end{cases}$$

where $h(x) = 1$ if $x \leq 0$, $h(x) = (1 - x)^{\frac{1}{3}}$ if $0 < x < 1$ and $h(x) = 0$ if $x \geq 1$.

Compute the critical time y_c

1 POINT

A 2

B 0

C -1

D 1

SOL: Our conservation law is of the form $u_y + c(u)u_x = 0$, for $c(u) = (u^3 + 1)$. The critical time of existence is given by the expression

$$y_c := -\inf \left\{ \frac{1}{(c(h(s)))_s} : s \in \mathbb{R}, (c(h(s)))_s < 0 \right\}.$$

Now, $c(h(s))$ is equal to 2 if $s \leq 0$, $2 - s$ if $0 < s < 1$ and 1 if $s \geq 1$. Therefore, the only interval where the function $(c(h(s)))_s$ does not vanish is when $s \in (0, 1)$, obtaining that

$$y_c = -\frac{1}{-1} = 1.$$

9. • *hyperbolic* if $\delta(L)(x_0, y_0) > 0$

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B Parabolic for $\Omega = \{x = 0, y \in \mathbb{R}\}$

C Elliptic for $\Omega = \{x < 0, -3 < y < 0\}$

D Hyperbolic for $\Omega = \{y > 0, x \in \mathbb{R}\}$

SOL:

The first reflex should be to find out what a, b and c are:

$$a = x^2, b = \frac{1}{2}xy, c = -y$$

Then write down the discriminant:

$$\delta(L) = b^2 - ac = \frac{1}{4}x^2y^2 + x^2y$$

Now we can just insert the conditions given (e.g. $x < a, y > b$) and see if the condition for the discriminant is fulfilled.

It is parabolic in $\Omega = \{y = -4, x \in \mathbb{R}\}$

10. (f) Let $u : D \rightarrow \mathbb{R}$ be the solution to the Laplace equation with Neumann boundary conditions

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B For all $\alpha, \beta \in \mathbb{R}, s.t. \beta = 0$

C For all $\alpha, \beta \in \mathbb{R}, s.t. 4\alpha + 3\beta = 0$

D For all $\alpha, \beta \in \mathbb{R}, s.t. 3\alpha + 4\beta = 0$

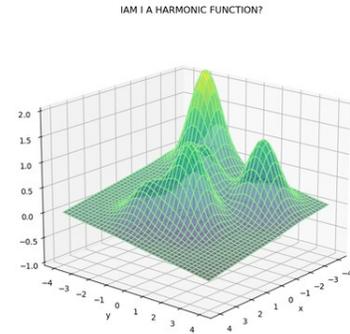
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 False

↳ Maximum principle!



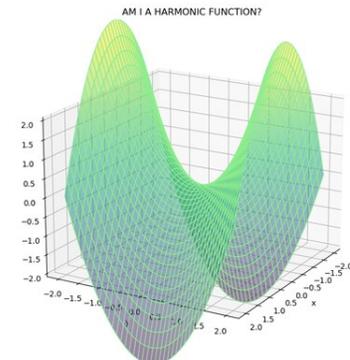
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"Can the pass be parametrized by a harmonic function?"

1 POINT

- True
 False

↳ Mean value principle, or guess that $u = x^2 - y^2$



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1 POINT

- A 3/4
 B 1/4
 C 1
 D 3/2

- i Maximum principle \rightarrow max on boundary. Maximised when x is big and the norm of the vector (x, y) is small, this is the case when $y = 0$ and $x = 1$ or 2 . Then $u = 1$