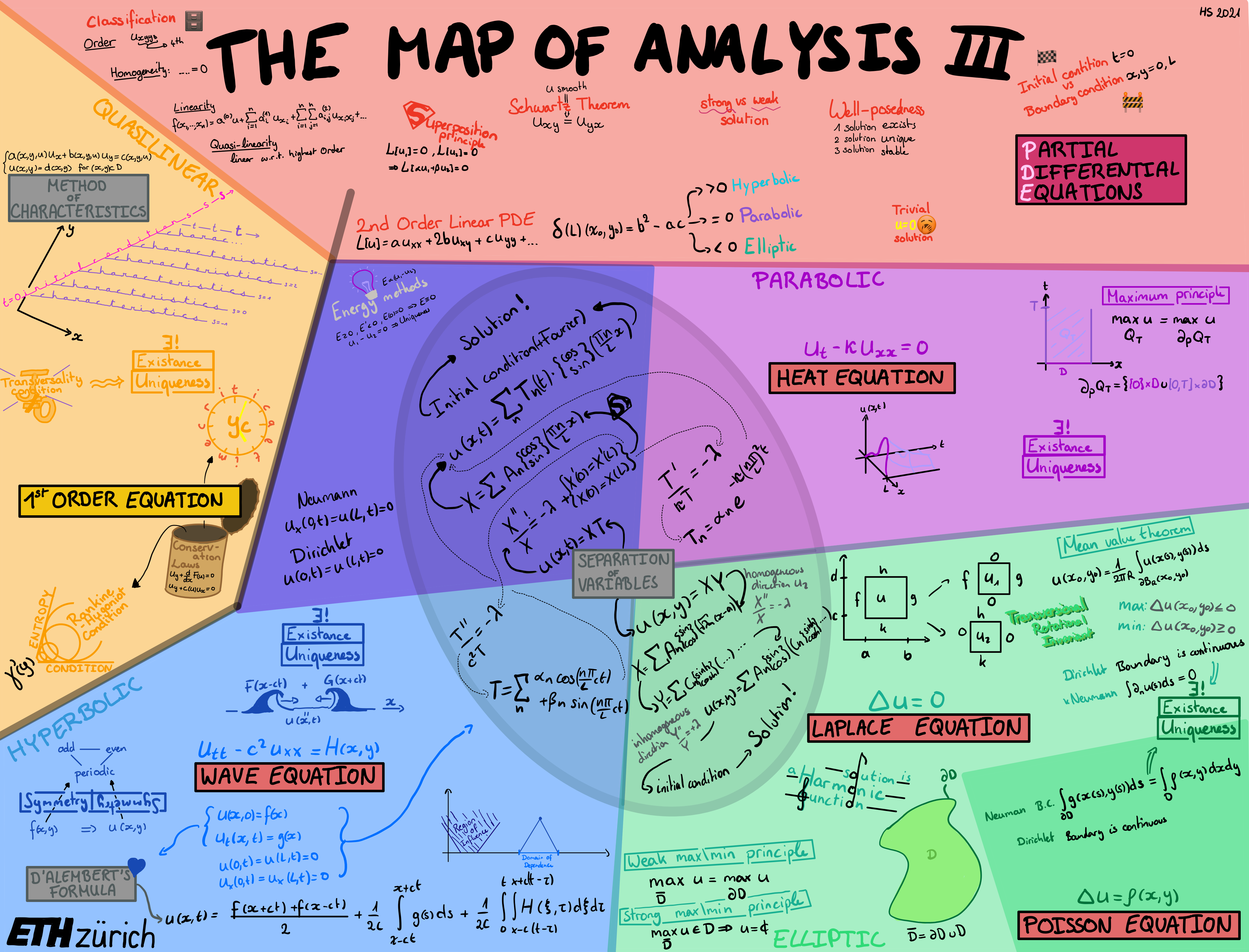
THE MAP OF ANALYSIS III
HS 2021
Classification
Order
Homogeneity: ... = 0
Linearity
Quasi-linearity
linear w.r.t. highest order
Superposition principle
$L[u_1]=0,\ L[u_2]=0 \Rightarrow L[\alpha u_1+\beta u_2]=0$
u smooth
Schwartz Theorem
$u_{xy}=u_{yx}$
strong vs weak solution
Well-posedness
1 solution exists
2 solution unique
3 solution stable
Initial condition t=0
vs
Boundary condition x,y=0,L
PARTIAL DIFFERENTIAL EQUATIONS
2nd Order Linear PDE
$L[u]=au_{xx}+2bu_{xy}+cu_{yy}+\dots$
$\delta(L)(x_0,y_0)=b^2-ac$
>0 Hyperbolic
=0 Parabolic
<0 Elliptic
Trivial u=0 solution
QUASILINEAR
METHOD OF CHARACTERISTICS
characteristics
t=0 initial condition
∃!
Existance
Uniqueness
Transversality condition ≠0
1st ORDER EQUATION
Conservation Laws
ENTROPY CONDITION
Rankine-Hugoniot Condition
Energy methods
Solution!
Initial condition + Fourier
Neumann
$u_x(0,t)=u(L,t)=0$
Dirichlet
$u(0,t)=u(L,t)=0$
SEPARATION OF VARIABLES
PARABOLIC
$u_t-ku_{xx}=0$
HEAT EQUATION
Maximum principle
$\max_{Q_T} u=\max_{\partial_p Q_T} u$
∃!
Existance
Uniqueness
Mean value theorem
max: $\Delta u(x_0,y_0)\le 0$
min: $\Delta u(x_0,y_0)\ge 0$
Translational Rotational Invariant
Dirichlet Boundary is continuous
Neumann
∃!
Existance
Uniqueness
$\Delta u=0$
LAPLACE EQUATION
homogeneous direction
inhomogeneous direction
Solution!
Initial condition
a Harmonic function
solution is
HYPERBOLIC
∃!
Existance
Uniqueness
$F(x-ct)+G(x+ct)$
odd — even
periodic
Symmetry
$u_{tt}-c^2u_{xx}=H(x,y)$
WAVE EQUATION
$u(x,0)=f(x)$
$u_t(x,t)=g(x)$
$u(0,t)=u(L,t)=0$
$u_x(0,t)=u_x(L,t)=0$
Region of Influence
Domain of Dependence
D'ALEMBERT'S FORMULA
$u(x,t)=\frac{f(x+ct)+f(x-ct)}{2}+\frac{1}{2c}\int_{x-ct}^{x+ct}g(s)ds+\frac{1}{2c}\int_0^t\int_{x-c(t-\tau)}^{x+c(t-\tau)}H(\xi,\tau)d\xi d\tau$
Weak max/min principle
Strong max/min principle
Neuman B.C.
Dirichlet Boundary is continuous
$\Delta u=\rho(x,y)$
POISSON EQUATION
ELLIPTIC
ETH zürich